

Analysis of scalar perturbations in cosmological models with a non-local scalar field

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Abstract

We develop the cosmological perturbations formalism in models with a single non-local scalar field originating from the string field theory description of the rolling tachyon dynamics. We construct the equation for the energy density perturbations of the non-local scalar field in the presence of the arbitrary potential and formulate the local system of equations for perturbations in the linearized model when both simple and double roots of the characteristic equation are present. We carry out the general analysis related to the curvature and entropy perturbations and consider the most specific example of perturbations when important quantities in the model become complex.

1 Introduction

Current cosmological observational data [1, 2] strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating dark energy (DE) component [3]. Recent results of WMAP [2] together with the data on Ia supernovae and galaxy clusters measurements, give the following bounds for the DE state parameter $w_{\text{DE}} = -1.02^{+0.14}_{-0.16}$. The present cosmological observations do not exclude a possibility that the DE with $w < -1$ exists, as well as an evolving DE state parameter w . Moreover, the recent analysis of the observation data indicates that the varying in time dark energy with the state parameter w_{DE} ,

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which crosses the cosmological constant barrier, gives a better fit than a cosmological constant (for details see reviews [4] and references therein).

Construction of a stable model with $w < -1$ is a challenge leading to the consideration of originally stable theories admitting the NEC violation in some limits. Recently a new class of cosmological models obeying this property which is based on the string field theory (SFT) [5] and the p -adic string theory [6] has been investigated a lot [7]–[26]. It is known that both the SFT and the p -adic string theory are UV-complete ones. Thus one can expect that resulting (effective) models should be free of pathologies.

Models originating from the SFT are distinguished by presence of specific non-local operators. The higher derivative terms in principle may produce the well known Ostrogradski instability [27] (see also [28, 8])¹. However the Ostrogradski result is related to higher than two but a finite number of derivatives. In the case of infinitely many derivatives it is possible that instabilities do not appear [19].

The SFT inspired cosmological models [7] are considered as models for dark energy (DE). The way of solving the Friedmann equations with a quadratic potential, by reducing them to the Friedmann equations with many free massive local scalar fields, has been proposed in [10, 12] (see also [25]). The obtained local fields satisfy the second order linear differential equations. In the representation of many scalar fields some of them are normal and some of them are phantom (ghost) ones [12, 25]. Cosmological models coming out from the SFT or the p -adic string theory are considered in application to inflation [15]–[22] to explain in particular appearance of non-gaussianities. For a more general discussion on the string cosmology and coming out of string theory theoretical explanations of the cosmological observational data the reader is referred to [31]. Other models obeying non-locality and their cosmological consequences are considered in [32, 33].

As a simplest model originating from SFT one can consider a theory with one scalar field whose kinetic operator is non-local. Equations for cosmological perturbations in such kind of model where the scalar field Lagrangian is quadratic covariantly coupled with Einstein gravity were derived in [23]. In the present paper we develop and improve that formalism accounting an arbitrary potential of the scalar field as well as the presence of double roots of the characteristic equation in the linearized model. We also carry out the general analysis of curvature and entropy perturbations and consider the most specific example of perturbations when characteristic quantities of the model become complex.

The paper is organized as follows. In Section 2 we describe the non-local non-linear SFT inspired cosmological model. In Section 3 we sketch the construction of background solutions in the linearized model and perturbation

¹Additional phantom solutions, obtained by the Ostrogradski method in some models can be interpreted as non-physical ones, because the terms with higher-order derivatives are regarded as corrections essential only at small energies below the physical cutoff [29, 30].

theory for models with non-local scalar field². In Section 4 we consider the perturbations in the case of two complex conjugate roots. In Section 5 we consider the case when the analytic function $\mathcal{F}(\alpha'\square)$, which defines the kinetic operator in the action, has double roots. In Section 6 we summarize the obtained results and propose directions for further investigations.

2 Model setup

2.1 Actions

We work in $(1+3)$ dimensions, the coordinates are denoted by Greek indices μ, ν, \dots running from 0 to 3. Spatial indexes are a, b, \dots and they run from 1 to 3. The four-dimensional action motivated by the string field theory is as follows [35, 10, 11]:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \frac{1}{\alpha' g_o^2} \left(\frac{1}{2} T \mathcal{F}_0(\alpha' \square) T - V_{int}(T) \right) - \Lambda_0 \right). \quad (1)$$

Here G_N is the Newtonian constant: $8\pi G_N = 1/M_P^2$, where M_P is the Planck mass, α' is the string length squared, g_o is the open string coupling constant. We use the signature $(-, +, +, +)$, $g_{\mu\nu}$ is the metric tensor, R is the scalar curvature, $\square = D^\mu \partial_\mu = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$ and D_μ being a covariant derivative, T is a scalar field primarily associated with the open string tachyon. The function $\mathcal{F}_0(\alpha' \square)$ may be not a polynomial manifestly producing thereby the non-locality. Fields are dimensionless while $[g_o] = \text{length}$. The potential $V_{int}(T)$, which is an open string tachyon self-interaction, does not have a quadratic term. It is convenient to introduce dimensionless coordinates $\bar{x}_\mu = x_\mu / \sqrt{\alpha'}$, the dimensionless gravitational constant $\bar{G}_N = G_N / \alpha'$, and the dimensionless coupling constant $\bar{g}_o = g_o / \sqrt{\alpha'}$. In the following formulae we always use dimensionless coordinates and parameters omitting bars over them.

Function \mathcal{F}_0 is assumed to be an analytic function of its argument, such that one can represent it by the convergent series expansion with real coefficients:

$$\mathcal{F}_0 = \sum_{n=0}^{\infty} f_n \square^n \quad \text{and} \quad f_n \in \mathbb{R}. \quad (2)$$

Equations of motion are

$$G_{\mu\nu} = \frac{8\pi G_N}{g_o^2} T_{\mu\nu}, \quad (3)$$

$$\mathcal{F}_0(\square) T = V'_{int}(T), \quad (4)$$

²For applications of other multi-field cosmological models and related technical aspects see for instance [34].

where $G_{\mu\nu}$ is the Einstein tensor, the energy-momentum (stress) tensor is as follows

$$\begin{aligned} T_{\mu\nu} = & \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_\mu \square^l T \partial_\nu \square^{n-1-l} T + \partial_\nu \square^l T \partial_\mu \square^{n-1-l} T - \right. \\ & \left. - g_{\mu\nu} \left(g^{\rho\sigma} \partial_\rho \square^l T \partial_\sigma \square^{n-1-l} T + \square^l T \square^{n-1-l} T \right) \right) - \\ & - g_{\mu\nu} \left(g_o^2 \Lambda_0 - \left(\frac{1}{2} T \mathcal{F}_0(\square) T - V_{int}(T) \right) \right). \end{aligned}$$

It is easy to check that the Bianchi identity is satisfied on-shell and for $\mathcal{F}_0 = f_1 \square + f_0$ the usual energy-momentum tensor for the massive scalar field is reproduced.

Let us emphasize that the potential of the field T is $V = -\frac{f_0}{2} T^2 + V_{int}(T)$. Let T_0 be an extremum of the potential V . One can linearize the theory in its neighborhood using $T = T_0 + \tau$. To second order in τ one gets the following action

$$S_2 = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \frac{1}{2g_o^2} \tau \mathcal{F}(\square) \tau - \Lambda \right), \quad (5)$$

where $\mathcal{F} = \mathcal{F}_0 - V(T_0)''$ and $\Lambda = \Lambda_0 + \frac{V(T_0)}{g_o^2}$. Equations (3) and (4) are valid for the latter action after the replacement $\mathcal{F}_0 \rightarrow \mathcal{F}$ and $\Lambda_0 \rightarrow \Lambda$ at $V_{int}(T) = 0$. Note that all Taylor series coefficients f_n , except f_0 , are the same for \mathcal{F}_0 and \mathcal{F} . Equation (4) now is

$$\mathcal{F}(\square) \tau = 0. \quad (6)$$

Non-local cosmological models of type (5) with

$$\mathcal{F}_{\text{sft}}(\square) = -\xi^2 \square + 1 - c e^{-2\square},$$

were previously considered in [11, 12, 17]³. Actions (1) and (5) are of the main concern of the present paper.

2.2 Background solutions construction in the linearized model

While solution construction in the full non-linear model (1) is not yet known the classical solutions to equations coming out the linearized action (5) were studied and analyzed in [10, 11, 12, 17, 20, 23, 25]. Thus, we just briefly notice the key points useful for purposes of the present paper.

³In [17] for example it has been shown that solving the non-local equations using the localization technique is fully equivalent to reformulating the problem using the diffusion-like partial differential equations. One can fix the initial data for the partial differential equation, using the initial data of the special local fields. This specifies initial data for a non-linear model, and these initial data can be (numerically) evolved into the full non-linear regime using the diffusion-like partial differential equation.

The main idea of finding solutions to the equations of motion is to start with equation (6) and to solve it, assuming the function τ is a sum of eigenfunctions of the d'Alembert operator:

$$\tau = \sum_i \tau_i, \text{ where } \square \tau_i = J_i \tau_i \text{ and } \mathcal{F}(J_i) = 0 \text{ for any } i = 1, \dots, N. \quad (7)$$

Hereafter we use N (which can be infinite as well) denoting the number of roots and omit it in writing explicit summation limits over i . Without loss of generality we assume that for any i_1 and $i_2 \neq i_1$ condition $J_{i_1} \neq J_{i_2}$ is satisfied. In this way of solving all the information is extracted from the roots of the *characteristic* equation $\mathcal{F}(J) = 0$. We can consider the solution τ as a general solution if all roots of \mathcal{F} are simple. The analysis is more complicated in the case of double roots [25]. We consider this case separately in Section 5.

In an arbitrary metric the energy–momentum tensor in (3) evaluated on such a solution is [23]

$$T_{\mu\nu} = \sum_i \mathcal{F}'(J_i) \left(\partial_\mu \tau_i \partial_\nu \tau_i - \frac{g_{\mu\nu}}{2} (g^{\rho\sigma} \partial_\rho \tau_i \partial_\sigma \tau_i + J_i \tau_i^2) \right) - g_{\mu\nu}^2 \Lambda. \quad (8)$$

The last formula is exactly the energy–momentum tensor of many free massive scalar fields. If $\mathcal{F}(J)$ has simple real roots, then positive and negative values of $\mathcal{F}'(J_i)$ alternate, so we can obtain phantom fields. Using formula (8) we obtain the Ostrogradski representation [27, 28] for action (5):

$$S_3 = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \Lambda - \sum_{i=1} \frac{\mathcal{F}'(J_i)}{2g_o^2} (g^{\mu\nu} \partial_\mu \tau_i \partial_\nu \tau_i + J_i \tau_i^2) \right).$$

one can see that S_3 is a local action if the number of roots N is finite.

2.3 Application to Friedmann–Robertson–Walker Universe

We stress that all the above formulae are valid for an arbitrary metric and the general solution. From now on, however, the only metric we will be interested in is the spatially flat Friedmann–Robertson–Walker (FRW) metric with the interval given by

$$ds^2 = -dt^2 + a^2(t) (dx_1^2 + dx_2^2 + dx_3^2) \quad (9)$$

where $a(t)$ is the scale factor, t is the cosmic time.

Background solutions for τ are taken to be space-homogeneous. The energy–momentum tensor in (3) in this metric can be written in the form of

a perfect fluid $T_\nu^\mu = \text{diag}(-\varrho, p, p, p)$, where

$$\begin{aligned}\varrho &= \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_t \square^l T \partial_t \square^{n-1-l} T + \square^l T \square^{n-l} T \right) - \\ &\quad - \frac{1}{2} T \mathcal{F}_0(\square) T + V_{int}(T) + g_o^2 \Lambda_0, \\ p &= \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_t \square^l T \partial_t \square^{n-1-l} T - \square^l T \square^{n-l} T \right) + \\ &\quad + \frac{1}{2} T \mathcal{F}_0(\square) T - V_{int}(T) - g_o^2 \Lambda_0.\end{aligned}\tag{10}$$

Obviously we can rewrite equations (3) in the canonical form:

$$3H^2 = 8\pi G \varrho, \quad \dot{H} = -4\pi G(\varrho + p),\tag{11}$$

where $G \equiv G_N/g_o^2$ is a dimensionless analog of the Newtonian constant, $H = \dot{a}/a$ and a dot denotes a derivative with respect to the cosmic time t . The consequence of (11) is the conservation equation:

$$\dot{\varrho} + 3H(\varrho + p) = 0.\tag{12}$$

which corresponds to the non-local field equation (4).

Some progress is possible in the linearized model when the metric is fixed to be of the FRW type. The individual equations in (7) in this metric read

$$\ddot{\tau}_i + 3H\dot{\tau}_i + J_i\tau_i = 0\tag{13}$$

The full system of equations of motion has the fixed points at $\tau_i = 0$ and $3H^2 = 3H_0^2 = 8\pi G_N \Lambda$, which is real at $\Lambda > 0$. Equations (13) together with Friedmann equations describe the late time evolution of the model with Lagrangian (1). This model possesses an approximate solution with all scalar fields tending to the minimum of the potential (i.e. $\tau_i \rightarrow 0$) and the Hubble parameter going to the constant. Such solution was constructed numerically and was proven to be a solution in [14]. Also the asymptotic form of this solution was derived in [10].

It is instructive to investigate the Lyapunov stability of the fixed point. Using formulae from [41], we come to conclusion the fixed point is asymptotically stable at

$$H_0 > 0, \quad \Re(J_i) < 0.\tag{14}$$

If at the fixed point $H_0 < 0$ or $\Re(J_i) > 0$ for some i , then this fixed point is unstable. At $\Re(J_i) = 0$ for some i or $H_0 = 0$ one can not use the Lyapunov theorem to analyse the stability of the fixed point. Note that the conditions (14) are sufficient for stability in not only the Friedmann–Robertson–Walker metric but also the Bianchi I metric [41]. In this paper we shall extend this analysis and consider the stability of the fixed point with respect to arbitrary perturbations.

To compute an approximate solution to (13) one starts with a constant $H = H_0$ and then computes the correction to H using Friedmann equations. Then the procedure can be iterated to compute higher corrections. It was proven in [10] such iteration does converge.

Solution to (13) with constant $H = H_0$ is obviously

$$\tau_i = \tau_{i+} e^{\alpha_{i+} t} + \tau_{i-} e^{\alpha_{i-} t} \quad (15)$$

where $\alpha_{i\pm} = \frac{3H_0}{2} \left(-1 \pm \sqrt{1 - \frac{4J_i}{9H_0^2}} \right)$. Considering τ_i we see that only one term in the solution converges when $t \rightarrow \infty$ in general (if both converge we select the slowest one). Let's assume it is the first one proportional to τ_+ constant. Then in order to pick the (slowest) convergent solution we put $\tau_- = 0$.⁴

The first correction to the constant Hubble parameter in case only decaying modes in τ_i are present is

$$H = H_0 + h = H_0 + h_0 \sum \tau_i^2. \quad (16)$$

Constant h_0 is not independent and is related with τ_{i+} . We note that h is of order τ_i^2 . The last expression is a good approximation for H in the asymptotic regime when $h \ll H_0$. Further one can find the scale factor to be

$$a = a_0 \exp \left(H_0 t + \frac{h_0}{2} \sum \frac{\tau_i^2}{\alpha_{i+}} \right). \quad (17)$$

3 Cosmological perturbations with single *non*-local scalar field

3.1 General analysis

Now we turn to the main problem of the present paper: derivation of cosmological perturbation equations in models with a non-local scalar field. We are focused on the scalar perturbations, because both vector and tensor perturbations exhibit no instabilities [37]. Scalar metric perturbations are given by four arbitrary scalar functions [36, 37]. Changing the coordinate system one can both produce fictitious perturbations and remove real ones. Natural way to distinct real and fictitious perturbations is introducing gauge-invariant variables, which are free of these complications and are equal to zero for a system without perturbations. There exist two independent gauge-invariant variables (the Bardeen potentials), which fully determine the scalar perturbations of the metric tensor [36, 37, 38, 40, 23]. To construct the perturbation equations one can use the longitudinal (conformal-Newtonian) gauge,

⁴Hereafter we adopt the rule $\sqrt{z^*} = \sqrt{z}^*$ meaning that the phase of the complex number runs in the interval $[-\pi, \pi)$ and for $z = r e^{i\sigma}$ the square root is $|\sqrt{r}| e^{i\sigma/2}$.

in which the interval (9) with scalar perturbations has the following form (in terms of the Bardeen potentials):

$$ds^2 = a(\eta)^2 \left(-(1 + 2\Phi)d\eta^2 + \delta_{ab}^{(3)}(1 - 2\Psi)dx^a dx^b \right) \quad (18)$$

where η is the conformal time related to the cosmic one as $a(\eta)d\eta = dt$. The the Bardeen potentials Φ and Ψ are as usually Fourier transformed with respect to the spatial coordinates x^a having thereby the following form: $\Phi(\eta, x^a) = \Phi(\eta, k)e^{ik_a x^a}$ and similar for Ψ . The obtained equations contain only gauge invariant variables, so they are valid in an arbitrary gauge.

Although the metric perturbations are defined in the conformal time frame in the sequel the cosmic time t will be used as the function argument and all the equations will be formulated with t as the evolution parameter.

To the background order energy density and pressure are given by (10). To the perturbed order one has

$$\begin{aligned} \delta \rho = & \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_t \delta(\square^l T) \partial_t \square^{n-1-l} T + \partial_t \square^l T \partial_t \delta(\square^{n-1-l} T) - \right. \\ & - 2\Phi \partial_t \square^l T \partial_t \square^{n-1-l} T + \\ & \left. + \delta(\square^l T) \square^{n-l} T + \square^l T \delta(\square^{n-l} T) \right) - \frac{1}{2g_o^2} (TV_{int}'' - V_{int}') \delta T, \end{aligned} \quad (19)$$

$$\begin{aligned} \delta p = & \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_t \delta(\square^l T) \partial_t \square^{n-1-l} T + \partial_t \square^l T \partial_t \delta(\square^{n-1-l} T) - \right. \\ & - 2\Phi \partial_t \square^l T \partial_t \square^{n-1-l} T - \\ & \left. - \delta(\square^l T) \square^{n-l} T - \square^l T \delta(\square^{n-l} T) \right) + \frac{1}{2g_o^2} (TV_{int}'' - V_{int}') \delta T, \end{aligned} \quad (20)$$

$$v^s = \frac{k}{a(\rho + p)} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_t \square^l T \delta(\square^{n-1-l} T), \quad (21)$$

$$\pi^s = 0. \quad (22)$$

where v^s gives the perturbed T_a^0 components of the stress-energy tensor and π^s is the anisotropic stress. Using the Einstein equations one gets that $\pi^s = 0$ is equivalent to $\Phi = \Psi$. The Bardeen potential Ψ is proportional to the gauge invariant total energy perturbation

$$\varepsilon \equiv \frac{\delta \rho}{\rho} + 3(1 + \omega)Hv^s \frac{a}{k} = - \frac{k^2}{4\pi G \rho a^2} \Psi. \quad (23)$$

The function ε is a solution of the following linear differential equation (see details in [23]):

$$\begin{aligned} & \ddot{\varepsilon} + H(2 + 3c_s^2 - 6w)\dot{\varepsilon} + \\ & + \left(\dot{H}(1 - 3w) - 15H^2 w + 9H^2 c_s^2 + \frac{k^2}{a^2} \right) \varepsilon + \frac{k^2}{a^2 \rho} \Delta = 0. \end{aligned} \quad (24)$$

Here $w = p/\rho$ is the equation of state parameter, $c_s^2 = \dot{p}/\dot{\rho}$ is the speed of sound, $k = \sqrt{k_a k^a}$ is the comoving wavenumber and

$$\begin{aligned} \Delta = & \delta p - \delta \varrho + (1 - c_s^2) \frac{a}{k} \dot{\varrho} v^s = \frac{(1 - c_s^2) \dot{\varrho}}{\varrho + p} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_t \square^l T \delta(\square^{n-1-l} T) - \\ & - \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\delta(\square^l T) \square^{n-l} T + \square^l T \delta(\square^{n-l} T) \right) + \frac{1}{g_o^2} (TV_{int}'' - V_{int}') \delta T. \end{aligned}$$

The latter quantity is identically zero for a local scalar field, i.e. in the case $\mathcal{F}(\square) = f_1 \square + f_0$. Therefore, $\Delta \neq 0$ is the attribute of the non-locality here.

For the linearized model (5) we can consider the background solution as given by (7) to obtain Δ in the more convenient form. To do this the following relation is useful

$$\delta(\square^n \tau) = \square^n \delta \tau + \sum_{m=0}^{n-1} \square^m (\delta \square) \square^{n-1-m} \tau. \quad (25)$$

Using (7) and the well-known formula

$$\sum_{m=0}^{n-1} x^m = \frac{x^n - 1}{x - 1},$$

one has

$$\delta(\square^n \tau) = \square^n \delta \tau + \sum_i \frac{\square^n - J_i^n}{\square - J_i} (\delta \square) \tau_i. \quad (26)$$

Perturbing the equation of motion for τ , one has

$$\delta(\mathcal{F} \tau) = \sum_{n=0}^{\infty} f_n \delta(\square^n \tau) = 0. \quad (27)$$

More explicitly this equation can be written as

$$\delta(\mathcal{F} \tau) = \mathcal{F} \sum_i \left(\frac{1}{\square - J_i} (\delta \square) \tau_i + \delta \tau_i \right) = 0 \quad (28)$$

where we have put $\delta \tau = \sum_i \delta \tau_i$.

It follows from (8) that if for some J_k we have $\tau_k = 0$ as a background solution, then $\delta \tau_k$, contributes only to the second order in the energy-momentum tensor perturbations. In this paper we consider perturbations only to the first order, and therefore for all $\tau_k = 0$ we can put $\delta \tau_k = 0$ without loss of generality. If \mathcal{F} has an infinite number of roots, but we select as a background the function τ , which includes only a finite number of τ_k , then only a finite number of perturbations $\delta \tau_k$ give contribution to the first

order perturbation equations, whereas in the second order all perturbations are important.

After some algebra one can get the following expression for Δ

$$\Delta = -\frac{2}{\varrho + p} \sum_{m,l} \mathcal{F}'(J_m) \mathcal{F}'(J_l) J_m \tau_m \dot{\tau}_m \dot{\tau}_l^2 \zeta_{ml}, \quad (29)$$

where $\zeta_{ij} = \frac{\delta\tau_i}{\dot{\tau}_i} - \frac{\delta\tau_j}{\dot{\tau}_j}$ satisfy the following set of equations

$$\begin{aligned} \ddot{\zeta}_{ij} + \left(3H + \frac{\ddot{\tau}_i}{\dot{\tau}_i} + \frac{\ddot{\tau}_j}{\dot{\tau}_j}\right) \dot{\zeta}_{ij} + \left(-3\dot{H} + \frac{k^2}{a^2}\right) \zeta_{ij} = \\ = \left(\frac{J_i \tau_i}{\dot{\tau}_i} - \frac{J_j \tau_j}{\dot{\tau}_j}\right) \left(\sum_m \frac{\mathcal{F}'(J_m) \dot{\tau}_m^2}{\varrho + p} (\dot{\zeta}_{im} + \dot{\zeta}_{jm}) + \frac{2}{1+w} \varepsilon\right). \end{aligned} \quad (30)$$

Equation (24) with the above derived Δ and equations (30) in a closed form describe the perturbations in the case of linearized model. Comparing these equation with the equation for perturbations in a system with many fields (see eqs. (82) and (85) in [38]) we see they do coincide. Thus perturbations become equivalent in the model with one free non-local scalar field and in the model with many local scalar fields. In our model, however, the quantity which should be considered as energy density perturbation is ε . Functions ζ_{ij} play auxiliary role and normally should not be given an interpretation.

3.2 Curvature and entropy perturbations

To understand better what is going on in the discussed type of models it is instructive to see how the curvature and entropy perturbations behave in our model having infinitely many scalar degrees of freedom (interesting results in analysing this parameters in case of many fields can be found for instance in [39]).

Comoving curvature perturbations can be expressed as

$$\mathcal{R} = \Psi - \frac{H}{\dot{H}} (\dot{\Psi} + H\Psi) \quad (31)$$

where Ψ is the Bardeen potential connected in turn with ε as (23). Entropy perturbations defined as $e = \delta p - c_s^2 \delta \varrho$ can be found as

$$\frac{e}{\varrho} = \varepsilon - (1 + c_s^2) \frac{a^2}{k^2} \Delta \quad (32)$$

Both quantities are gauge invariant and play crucial role in computing various spectral indexes.

In order to figure out the behavior for the curvature and entropy perturbations it is enough to find out the behavior of only two functions: ε and Δ . Such a generic form in formulae (31) and (32) persists because the theory

can be written in the local form. It is nevertheless obvious that it would be difficult to reach similar results in a general case, when $V_{int} \neq 0$ and equation (3) is not linear in T . Moreover, it is not transparent that one can make a significant progress for a general operator \mathcal{F} even in the linearized model.

There are several specific situations in which we can outline the strategy of the further analysis.

- The first one is already mentioned above and refers to the background configuration where only the finite number of scalar fields is excited. Then in the linear perturbations we will have no impact of the perturbation modes related to the zero background fields. This statement is obvious having in mind that the local action is quadratic and diagonal in scalar fields.
- The second situation corresponds to particular form of the function \mathcal{F} such that its roots form a sequence J_i and one can arrange them such that $J_i - J_{i+1} \rightarrow 0$ when i tends to infinity. Then in the asymptotic regime where solutions for the scalar fields are dumped plane waves (15) one gets the factor in the RHS of equation (30) tending to zero and equations for corresponding ζ_{ii+1} become homogeneous. In this case one can easily solve this equation which now reads

$$\ddot{\zeta}_{ii+1} + c\dot{\zeta}_{ii+1} + \left(-3\dot{H} + \frac{k^2}{a^2}\right)\zeta_{ii+1} = 0. \quad (33)$$

Here we assumed the coefficient in front of the first derivative to be a constant while the background solutions are the planewaves. Two modes come out when the time grows

$$\zeta_{ii+1} \approx C_1 + C_2 e^{-ct} \quad (34)$$

and therefore we see that there is a decaying or growing mode depending on the sign of c . This constant in the asymptotic regime is given by

$$c = 3H_0 - \left(\sqrt{J_i} + \sqrt{J_{i+1}}\right). \quad (35)$$

Recall that the stability of the asymptotic solution requires $\Re(J_i) < 0$ (see (14)) and it guarantees that $c > 0$ and perturbations vanish. Even though it is not the final answer and one still has to solve coupled equations the claim is that only a finite number of ζ_{ij} are really coupled with ε while other functions ζ_{ij} just produce the inhomogeneity in equation for ε (24). Mathematically we can put all this decoupled modes to be zero but than the problem becomes completely equivalent to the previous case with a finite number of fields in the play. Here, however, we keep trace of other fields even though they are effectively decoupled from the system of equations.

One example of the operator giving such a behavior is

$$\mathcal{F}(J) = \alpha - e^{\beta J} \quad (36)$$

Roots are given by

$$J_k = \frac{1}{\beta}(\log(\alpha) + 2\pi i k) \quad (37)$$

Resummation in Δ (29) for all the components which are effectively decoupled (assuming this happens from some $k = k_0$) gives

$$\Delta_0 = -\frac{2C_2}{\varrho + p} \frac{e^{-ct}}{1 + e^{-ct}} \quad (38)$$

where C_2 and c are from eq. (35) when $i = k_0$.

- The third situation is when the values of $\sqrt{J_i}$ are equidistant meaning $\sqrt{J_{i+1}} - \sqrt{J_i} = c_J$. Then equation (30) in the asymptotic regime becomes

$$\begin{aligned} & \ddot{\zeta}_{i+1i} + \left(3H + 2\sqrt{J_i} + c_J\right) \dot{\zeta}_{i+1i} + \left(-3\dot{H} + \frac{k^2}{a^2}\right) \zeta_{i+1i} = \\ & = c_J \left(\sum_m \frac{\mathcal{F}'(J_m) \dot{\tau}_m^2}{\varrho + p} \left(\dot{\zeta}_{i+1m} + \dot{\zeta}_{im} \right) + \frac{2}{1+w} \varepsilon \right). \end{aligned} \quad (39)$$

The advantage with respect to the general situation is that the inhomogeneous part of this equation is universal for any number i meaning that we can construct homogeneous equations considering the difference of the latter equations for some i and j .

It is important to say that all these cases even giving some insight into the problem are not very simple to analyze. We hope to address this issue in the future analysis of models of this type.

4 Complex roots J in the linearized model

4.1 One pair of complex conjugate roots $J_1 = J$ and $J_2 = J^*$. The background

If a complex number J is a root of \mathcal{F} , then J^* is a root of \mathcal{F} as well. System (11) becomes:

$$\begin{aligned} H^2 &= \frac{8}{3} \pi G_N \left[\frac{\mathcal{F}'(J)}{2g_o} (\dot{\tau}^2 + J\tau^2) + \frac{\mathcal{F}'(J^*)}{2g_o} (\dot{\tau}^{*2} + J^*\tau^{*2}) + \Lambda \right], \\ \dot{H} &= -\frac{4\pi G_N}{g_o} \left[\mathcal{F}'(J) \dot{\tau}^2 + \mathcal{F}'(J^*) \dot{\tau}^{*2} \right]. \end{aligned} \quad (40)$$

In terms of real fields ϕ and ψ such that $\tau = \phi + i\psi$, $\tau^* = \phi - i\psi$, we get the following kinetic term:

$$E_k = \frac{\mathcal{F}'(J)}{2g_o} \dot{\tau}^2 + \frac{\mathcal{F}'(J^*)}{2g_o} \dot{\tau}^{*2} = \frac{d_r}{g_o} (\dot{\phi}^2 - \dot{\psi}^2) + 2 \frac{d_i}{g_o} \dot{\phi} \dot{\psi}, \quad (41)$$

where $d_r = \Re(\mathcal{F}'(J))$ and $d_i = \Im(\mathcal{F}'(J))$. In the case $d_i \neq 0$ E_k has a nondiagonal form. To diagonalize kinetic term we make the following transformation:

$$\chi = \phi + C_1 \psi, \quad \nu = -C_1 \phi + \psi, \quad C_1 \equiv \frac{d_r + \sqrt{d_r^2 + d_i^2}}{\sqrt{d_i}}.$$

In terms of χ and ν system (40) has the following form:

$$\begin{aligned} H^2 &= \frac{8}{3} \pi G_N \left[\frac{C}{2g_o^2} (\dot{\nu}^2 - \dot{\chi}^2 + J_r(\nu^2 - \chi^2) + 2J_m \nu \chi) + \Lambda \right], \\ \dot{H} &= \frac{4\pi G_N C}{g_o^2} (\dot{\chi}^2 - \dot{\nu}^2), \end{aligned} \quad (42)$$

where $J_r = \Re(J)$, $J_m = \Im(J)$, $C = \frac{d_i^2(d_r^2 + d_i^2)(d_r + \sqrt{d_r^2 + d_i^2})}{d_r^2 + d_i^2 + d_r \sqrt{d_r^2 + d_i^2}}$. So, in the case of two complex conjugated roots we get a quintom model (for details of quintom models see reviews [4]).

What is interesting (but not surprising, though) one cannot have non-interacting fields passing to the real components. Precisely, fields will be quadratically coupled in the Lagrangian. It means that the usual intuition about field properties (like signs of coefficients in front the kinetic term or the mass term) may not work.

Following the method outlined in Section 2.3 we find the asymptotic solution for the scalar fields with constant $H = H_0$ to be

$$\tau = \tau_+ e^{\alpha_+ t} + \tau_- e^{\alpha_- t}, \quad \tau^* = \tau_+^* e^{\alpha_+^* t} + \tau_-^* e^{\alpha_-^* t} \quad (43)$$

where $\alpha_{\pm} = \frac{3H_0}{2} \left(-1 \pm \sqrt{1 - \frac{4J}{9H_0^2}} \right)$. We assume the first term proportional to τ_+ does converge and put $\tau_- = 0$. Further we define $\tau_+ \equiv \tau_0$ and $\alpha_+ \equiv \alpha$.

The first correction to the constant Hubble parameter and to the scale factor in case only decaying modes in τ are present gives

$$H = H_0 + h = H_0 + h_0 (\tau^2 + \tau^{*2}). \quad (44)$$

and

$$a = a_0 \exp \left(H_0 t + \frac{h_0}{2} \left(\frac{\tau^2}{\alpha} + \frac{\tau^{*2}}{\alpha^*} \right) \right). \quad (45)$$

4.2 Cosmological perturbations in the neighborhood of the solution with complex masses

Configurations with a single scalar field were widely studied and those appearing in the non-local models do not have any distinguished properties. Roughly speaking configurations with many scalar fields were explored as well but we have here new models featuring complex masses and complex coefficients in front of the kinetic terms. As it was stressed above there is no problem with this for the physics of our models while properties of such models, in particular the cosmological perturbations with such scalar fields were not studied in depth. Thus we focus on perturbations in the configuration with complex roots J . The simplest case is one pair of complex conjugate roots where the background quantities were derived in previous Subsection.

First we note that the only function ζ_{ij} is ζ_{12} which we shall denote ζ . Thus, there are only two equations in the system. We focus on the asymptotic regime $h \ll H_0$ and after some algebra one arrives to the following system of equations

$$\begin{aligned} (\varrho + p) \left(\ddot{\zeta} + (3H_0 + \alpha + \alpha^*) \dot{\zeta} + \left(-3\dot{H} + \frac{k^2}{a_0^2} e^{-2H_0 t} \right) \zeta \right) = \\ = \left(\frac{J}{\alpha} - \frac{J^*}{\alpha^*} \right) \left(\left[\mathcal{F}'(J^*) \alpha^{*2} \tau^{*2} - \mathcal{F}'(J) \alpha^2 \tau^2 \right] \dot{\zeta} + 2g_o^2 \Lambda \varepsilon \right), \end{aligned} \quad (46)$$

$$\begin{aligned} \ddot{\varepsilon} + \dot{\varepsilon} H_0 (8 + 3c_s^2) + \varepsilon \left(15H_0^2 + 9H_0^2 c_s^2 + \frac{k^2}{a_0^2} e^{-2H_0 t} \right) = \\ = \frac{2k^2 \mathcal{F}'(J) \mathcal{F}'(J^*) \alpha^2 \alpha^{*2} \tau_0^2 \tau_0^{*2}}{(\varrho + p) a_0^2 g_o^2 \Lambda} \left(\frac{J}{\alpha} - \frac{J^*}{\alpha^*} \right) e^{2(-H_0 + \alpha + \alpha^*)t} \zeta. \end{aligned} \quad (47)$$

where we should use

$$\begin{aligned} \dot{H} &= 2h_0 \left(\tau^2 \alpha + \tau^{*2} \alpha^* \right), \\ \rho + p &= \mathcal{F}'(J) \tau^2 \alpha^2 + \mathcal{F}'(J^*) \tau^{*2} \alpha^{*2}, \\ c_s^2 &= \frac{\mathcal{F}'(J) \alpha \tau^2 (\alpha^2 - J) + \mathcal{F}'(J^*) \alpha^* \tau^{*2} (\alpha^{*2} - J^*)}{\mathcal{F}'(J) \alpha \tau^2 (\alpha^2 + J) + \mathcal{F}'(J^*) \alpha^* \tau^{*2} (\alpha^{*2} + J^*)}. \end{aligned}$$

The latter system of equations is ready to be solved numerically but in order to get some insight in what is going on it is instructive to make some assumptions about the value J . This makes some analytic progress possible.

We recall the SFT origin of the model. Practically this means that values of J are determined with the string scales while H_0 is expected to be much smaller. Therefore, it is natural to assume that $|\sqrt{J}| \gg H_0$. This implies $\alpha \approx i\sqrt{J}$. Using the explicit expression for $\tau_1 = \tau_0 e^{\alpha t} \approx \tau_0 e^{i\sqrt{J}t}$, representing $\alpha = x/2 + iy/2$ and introducing $\chi = i \frac{8\pi G}{3\Lambda} |\alpha^2| |\tau_0^2| e^{xt} \zeta$ the equations of interest

can be written as

$$\begin{aligned} & \cos(yt)\ddot{\chi} + 2(\sqrt{x^2 + y^2} \sin(yt + \sigma_b) - x \cos(yt))\dot{\chi} + \\ & + \left(\cos(yt) \left(6h_0 \sqrt{x^2 + y^2} e^{x(t-t_0)} \sin(y(t-t_0) - \sigma_b) + \frac{k^2}{a_0^2} + x^2 \right) - \right. \\ & \left. - 2x \sqrt{x^2 + y^2} \sin(yt + \sigma_b) \right) \chi = -2y\varepsilon \end{aligned} \quad (48)$$

$$\begin{aligned} & \cos(yt)\ddot{\varepsilon} + 2\sqrt{x^2 + y^2} \sin(yt - \sigma_b)\dot{\varepsilon} + \\ & + 3H_0 \sqrt{\left(\frac{k^2}{3a_0^2 H_0} - x \right)^2 + y^2} \cos(yt - \sigma_c) \varepsilon = \frac{2k^2 y}{a_0^2} \chi. \end{aligned} \quad (49)$$

where all the constant coefficients are real, ε and χ are real, t_0 is expected to be negative and

$$\sigma_b = \arcsin \frac{x}{\sqrt{x^2 + y^2}}, \quad \sigma_c = \arcsin \frac{y}{\sqrt{\left(\frac{k^2}{3a_0^2 H_0} - x \right)^2 + y^2}}.$$

The most alarming points of the evolution are $yt = \frac{\pi}{2} + n\pi$ where the coefficients in front of second derivatives become zero. Numeric integration may hit problems at these points if the precision is not very high. In the neighborhood of these points one has

$$t\ddot{\chi} - 2\dot{\chi} + 2x\chi = 2\varepsilon \quad (50)$$

$$t\ddot{\varepsilon} - 2\dot{\varepsilon} - 3H_0\varepsilon = \frac{2k^2}{a_0^2} \chi. \quad (51)$$

For negative x the solution for ε around $t = 0$ is $\varepsilon = \varepsilon_0 + \varepsilon_1 t + \dots$ meaning that these points are not singular for the above system of equations.

A typical behavior for the function ε is dumped oscillations depicted in Fig. 1. Such a behavior does not depend on the wavenumber meaning

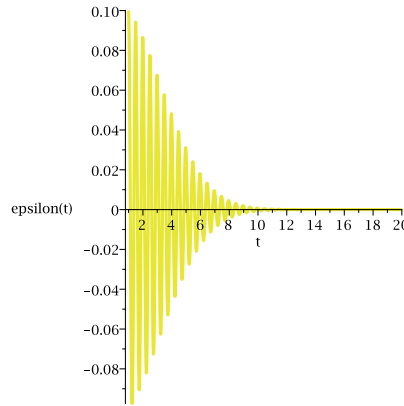


Figure 1: Typical behavior of the function ε .

that perturbations with complex conjugate scalar fields do vanish. This is different from usual models with real scalar fields where different regimes exist and most likely growing modes are present.

Application of the curvature and entropy perturbation analysis (formulae (31) and (32)) shows that both quantities decay exponentially in the presence of a pair of complex conjugate roots. Moreover accounting of the effect of possible other modes which correspond to other roots with the property that these modes are decoupled from the full system of perturbation equations (as in (33)–(38)) does not change qualitatively the result. More comprehensive analysis on what happens when complex roots which are significant for perturbations are present can be found in [44].

5 The linearized model with double roots

Let us consider an analytical function $\mathcal{F}(J)$, which has simple roots J_i and double roots \tilde{J}_k , and the function

$$\tau_B = \sum_{i=1}^{N_1} \tau_i + \sum_{k=1}^{N_2} \tilde{\tau}_k, \quad \text{where} \quad (\square - J_i)\tau_i = 0, \quad (\square - \tilde{J}_k)^2 \tilde{\tau}_k = 0. \quad (52)$$

The fourth order differential equation $(\square - \tilde{J}_k)(\square - \tilde{J}_k)\tilde{\tau}_k = 0$ is equivalent to the following system of the second order equations:

$$(\square - \tilde{J}_k)\tilde{\tau}_k = \sigma_k, \quad (\square - \tilde{J}_k)\sigma_k = 0.$$

It is convenient to write $\square^m \tilde{\tau}_k$ in terms of $\tilde{\tau}_k$ and σ_k :

$$\square^m \tilde{\tau}_k = \tilde{J}_k^m \tilde{\tau}_k + m \tilde{J}_k^{m-1} \sigma_k. \quad (53)$$

The energy–momentum tensor, which corresponds to τ_B , has the following form [25]:

$$T_{\mu\nu}(\tau_B) = \sum_{i=1}^{N_1} T_{\mu\nu}(\tau_i) + \sum_{k=1}^{N_2} T_{\mu\nu}(\tilde{\tau}_k), \quad (54)$$

where

$$\begin{aligned} T_{\mu\nu}(\tau) &= \frac{1}{g_o^2} \left(E_{\mu\nu}(\tau) + E_{\nu\mu}(\tau) - g_{\mu\nu} (g^{\rho\sigma} E_{\rho\sigma}(\tau) + W(\tau)) \right), \\ E_{\mu\nu}(\tau_i) &= \frac{\mathcal{F}'(J_i)}{2} \partial_\mu \tau_i \partial_\nu \tau_i, \quad W(\tau_i) = \frac{J_i \mathcal{F}'(J_i)}{2} \tau_i^2, \\ E_{\mu\nu}(\tilde{\tau}_k) &= \frac{\mathcal{F}''(\tilde{J}_k)}{4} (\partial_\mu \tilde{\tau}_k \partial_\nu \sigma_k + \partial_\nu \tilde{\tau}_k \partial_\mu \sigma_k) + \frac{\mathcal{F}'''(\tilde{J}_k)}{12} \partial_\mu \sigma_k \partial_\nu \sigma_k, \\ W(\tilde{\tau}_k) &= \frac{\tilde{J}_k \mathcal{F}''(\tilde{J}_k)}{2} \tilde{\tau}_k \sigma_k + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right) \sigma_k^2. \end{aligned}$$

Formulae (52) and (54) generalize formulae (7) and (8) for the functions $\mathcal{F}(J)$ with both simple, and double roots. Using (25), (53) and

$$\sum_{m=0}^{n-1} mx^{m-1} = \frac{d}{dx} \sum_{m=0}^{n-1} x^m = \frac{d}{dx} \left(\frac{1-x^n}{1-x} \right) = \frac{(n-1)x^n - nx^{n-1} + 1}{(1-x)^2},$$

we get

$$\begin{aligned} \delta(\square^n \tau_B) &= \square^n (\delta \tau_B) + \sum_{m=0}^{n-1} \square^m (\delta \square) \square^{n-1-m} \tau_B = \\ &= \square^n (\delta \tau_B) + \sum_{i=1}^{N_1} \frac{\square^n - J_i^n}{\square - J_i} (\delta \square) \tau_i + \sum_{k=1}^{N_2} \left[\frac{\square^n - \tilde{J}_k^n}{\square - \tilde{J}_k} (\delta \square) \tilde{\tau}_k + \right. \\ &\quad \left. + \frac{\tilde{J}_k (\square^n + (n-1)\tilde{J}_k^n - n\tilde{J}_k^{n-1}\square)}{(\square - \tilde{J}_k)^2} (\delta \square) \sigma_k \right]. \end{aligned} \quad (55)$$

To formulate the perturbation equations we note that from (54) one finds

$$\varrho(\tau_B) - p(\tau_B) = 2 \left(\sum_{i=1}^{N_1} W(\tau_i) + \sum_{k=1}^{N_2} W(\tilde{\tau}_k) \right),$$

so that,

$$\begin{aligned} \delta \varrho(\tau_B) - \delta p(\tau_B) &= 2 \sum_{i=1}^{N_1} J_i \mathcal{F}'(J_i) \tau_i \delta \tau_i + \\ &+ \sum_{k=1}^{N_2} \left[\tilde{J}_k \mathcal{F}''(\tilde{J}_k) (\sigma_k \delta \tilde{\tau}_k + \tilde{\tau}_k \delta \sigma_k) + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{3} + \mathcal{F}''(\tilde{J}_k) \right) \sigma_k \delta \sigma_k \right] \end{aligned}$$

and

$$\begin{aligned} v^s(\tau_B) &= \frac{k}{a(\varrho+p)} \left(\sum_{i=1}^{N_1} J_i \mathcal{F}'(J_i) \dot{\tau}_i \delta \tau_i + \right. \\ &\quad \left. + \sum_{k=1}^{N_2} \left[\frac{\mathcal{F}''(\tilde{J}_k)}{2} (\dot{\sigma}_k \delta \tilde{\tau}_k + \dot{\tilde{\tau}}_k \delta \sigma_k) + \frac{\mathcal{F}'''(\tilde{J}_k)}{6} \dot{\sigma}_k \delta \sigma_k \right] \right), \end{aligned} \quad (56)$$

where

$$\varrho(\tau_B) + p(\tau_B) = 2 \left(\sum_{i=1}^{N_1} E_{00}(\tau_i) + \sum_{k=1}^{N_2} E_{00}(\tilde{\tau}_k) \right).$$

Then for one double root we obtain

$$\begin{aligned} \Delta(\tilde{\tau}_k) &= \frac{\mathcal{F}''(\tilde{J}_k)}{6} \left\{ 3\tilde{J}_k \mathcal{F}''(\tilde{J}_k) \left(\sigma_k \dot{\tilde{\tau}}_k^2 \delta \sigma_k + \dot{\sigma}_k^2 \tilde{\tau}_k \delta \tilde{\tau}_k \right) + \right. \\ &\quad \left. + 3\mathcal{F}''(\tilde{J}_k) \sigma_k \dot{\sigma}_k^2 \delta \tilde{\tau}_k + \tilde{J}_k \mathcal{F}'''(\tilde{J}_k) \sigma_k \dot{\sigma}_k \dot{\tilde{\tau}}_k \delta \sigma_k \right\}. \end{aligned} \quad (57)$$

Note that $\Delta(\tilde{\tau}_k) \neq 0$, because $\mathcal{F}''(\tilde{J}_k) \neq 0$.

After the diagonalization⁵ of the kinetic part of the energy–momentum tensor, one can use the general formulae for perturbations in cosmological

⁵Explicit formulae are given in [25]

models with many scalar fields [38] and get the closed system of equations for perturbations. So, we can conclude that in the case of the function $\mathcal{F}(\square)$ with both simple and double roots we get the system of local equations. We plan to consider deeper the case of the function $\mathcal{F}(\square)$ with double roots in our forthcoming paper.

6 Summary and outlook

The main results of this paper are the construction of the perturbation equations in non-local models with one scalar field and arbitrary potential and consideration of the very intriguing example of perturbations. Namely, using the possibility to construct a local equivalent model with many scalar fields we find out that masses of these local fields may easily become complex and such a case constitutes the above-mentioned example. The characteristic feature of the present setup is that all the local fields in fact are not physical and play a role of auxiliary functions introduced for the reduction of the complicated non-local problem to a known one. As it was noted in [10, 11], for a very wide class of the SFT inspired models the local counterpart is not yet studied. Looking strange such configurations do not produce a problem for the model since they are not physical quantities.

Perturbation equations for this local model are (24) and (30) where only $N - 1$ functions ζ_{1j} are independent. The discussion on how a cosmological constant can be generated during the tachyon evolution is presented in [7, 10]. We note that perturbations in a quintom model very close to our setup with a phantom field without potential and an ordinary scalar field with quadratic potential were studied in [42]. Perturbations in models with many scalar fields were studied in literature considering various cosmological scenarios [38, 43].

In the present paper we have worked the indicative example where two scalar fields with complex conjugate masses are present. We have demonstrated numerically that in the case $|\sqrt{J}| \gg H_0$ the gauge invariant energy density perturbation associated with the matter sector does decay in all wavelength regimes in contrary to ordinary scalar field models. The general case of complex masses deserves deeper investigation and is partially considered in [44].

Moreover we singled out configurations when really an infinite set of scalar fields may present but on the other hand there is a chance to analyze in full the system of equations for perturbations. These are configurations when $\sqrt{J_{i+1}} - \sqrt{J_i} \rightarrow 0$ and when $\sqrt{J_{i+1}} - \sqrt{J_i} = c_J = \text{const.}$ This should definitely help in understanding how important quantities like curvature perturbations and entropy perturbations behave. Also the case of double roots is addressed and it is shown that one can again formulate the local equivalent theory and build a closed system of equations. However, the deep analysis of all these regimes seems to be rather involved and therefore we put it as still open

question to be addressed separately.

Looking further it is interesting to consider perturbations in other non-local models coming from the SFT. For instance, models where open and closed string modes are non-minimally coupled may be of interest in cosmology. An example of the classical solution is presented in [45]. Furthermore it should be possible to extend the formalism presented in this paper to other models involving non-localities like modified gravity setups [32, 33].

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